Differentiation and integration:

- $\frac{d}{dx}(x^n) = nx^{n-1}$
- $\frac{d}{dx}(\ln x) = \frac{1}{x}$
- $\frac{d}{dx}(e^{x}) = e^x$
- $\frac{d}{dx}(a^{x}) = a^x . lna$
- $\frac{d}{dx}(f(x))^n = n\big(f(x)\big)^{n-1} . f'(x)$
- $\frac{d}{dx}(\ln f(x)) = \frac{1}{f(x)} . f'(x)$
- $\frac{d}{dx} e^{f(x)} . f'(x)$
- $\frac{d}{dx} a^{f(x)} = a^{f(x)} . lna . f'(x)$
- $\frac{d}{dx}\big(f_1(x) . f_2(x)\big) = f_1(x) . f_2{}'(x) + f_2(x) . f_1{}'(x)$
- $\frac{d}{dx}\left(\frac{f_1(x)}{f_2(x)}\right) = \frac{f_2(x).f_1'^{(x)} - f_1(x).f_2{}'(x)}{(f_2(x))^2}$

Find the derivative of each of the following:

1. f(x) = x^5

2. f(x) = $(5x)^6$

3. f(x) = e^{3x}

4. $f(x) = \frac{1}{5x}$

5. $f(x) = 3^x$

6. $f(x) = 5^{2x}$

7. $f(x) = 7x.e^{3x}$

8. $f(x) = \frac{4x}{2x+5}$

Integration:

- $\int x^n dx = \frac{x^{n+1}}{n+1} + C$

- $\int \left(f(x)\right)^n . f'(x) dx = \frac{[f(x)]^{n+1}}{n+1} + C$

- $\int \frac{1}{x} dx = \ln|x| + C$

- $\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + C$

- $\int e^x dx = e^x + C$

- $\int e^{f(x)} . f'(x) dx = e^{f(x)} + C$

- $\int a^x dx = \frac{a^x}{\ln a} + C$

- $\int a^{f(x)} f'(x) dx = \frac{a^{f(x)}}{\ln a} + C$

Some example:

Find the integration of the following functions.

for all the limits are from a to b.

1. $f(x) = x^4$ $\qquad \frac{1}{5}\,[\,b^5 - a^5]$
2. $f(x) = \; 6x^3$ $\qquad \frac{3}{2}[b^4 - a^4]$
3. $f(x) = e^{4x}$ $\qquad \frac{1}{4}[e^{4b} - e^{4a}]$
4. $f(x) = 2x.\,e^{3x^2}$ $\qquad \frac{1}{3}[e^{3b^2} - e^{3a^2}]$

$$\int \boldsymbol{f_1(x).\,f_2(x)\;dx}$$

$$= \boldsymbol{f_1(x).\int f_2(x)dx}$$

$$- \int \boldsymbol{f_2(x).\,f'_1(x)dx}$$

This rule is called integration by parts.

e.g. $\qquad f(x) = \frac{x}{2}e^x$

$=\int_a^b \frac{x}{2}.\,e^x dx$

Here: $f_1(x) = \frac{x}{2}$ $\qquad$ *and* $\qquad f_2(x) = e^x$

So according to the formula,

$$= \frac{x}{2} . \int_a^b e^x dx - \int_a^b e^x \left(\frac{d}{dx}\left(\frac{x}{2}\right)\right) dx$$

$$= \frac{x}{2} e^x \Big|_a^b - \frac{1}{2} \int_a^b e^x \, dx$$

$$= \frac{b}{2} e^b - \frac{a}{2} e^a - \frac{1}{2} [e^x|_a^b]$$

$$= \frac{b}{2} e^b - \frac{a}{2} e^a - \frac{1}{2} [e^b - e^a]$$

Gamma Function

- $\Gamma n = \int_0^{\infty} x^{n-1}\, e^{-x} dx$
- Γn = (n-1)! = (n-1) $\Gamma(n-1)$
- $\Gamma \frac{1}{2} = \sqrt{\Pi}$
- 0! = 1
- 1! = 1
- n! = n x (n-1) x (n-2) x (n-3) x (n-4) x --------x 3 x 2 x 1
- n! = n(n-1)!

➢ Examples:

1. $\int_0^{\infty} x^5 . e^{-x}\, dx = \int_0^{\infty} x^{5+1-1}\, e^{-x} dx$

$= \Gamma 6 = 5!$

2. $\int_0^{\infty} x^4\, e^{-3x}\, dx = \frac{1}{3^5} \int_0^{\infty} (3x)^4\, e^{-3x} . 3\, dx$

$= \frac{1}{3^5} \int_0^{\infty} (y)^{4+1-1}\, e^{-4}$ (let 3x =y)

$= \frac{1}{3^5} \Gamma(5) = \frac{4!}{3^5}$

3. $$\int_0^\infty y^2 . e^{-\frac{y}{5}}\, dy = 5^3 \int_0^\infty \left(\frac{y}{5}\right)^2 . e^{-\left(\frac{y}{5}\right)} . \frac{1}{5} . dy$$

$$= u^2 \int_0^\infty u^2 . e^{-u}\, du$$

$$= 5^3\, \Gamma 3 = \mathbf{5^3 \times 2!}$$

4. $$\int_0^\infty u^{\frac{1}{2}} . e^{-\frac{u}{3}}\, du = 3^{\frac{3}{2}} \int_0^\infty \left(\frac{u}{3}\right)^{\frac{1}{2}} . e^{-\frac{u}{3}} . \frac{1}{3}\, du$$

$$= 3^{\frac{3}{2}} \int_0^\infty x^{\frac{1}{2}} . e^{-x}\, dx \qquad \left(let\ \frac{u}{3} = x\right)$$

$$= 3^{\frac{3}{2}} \int_0^\infty x^{\frac{3}{2} - 1} . e^{-x}\, dx$$

$$= 3^{\frac{3}{2}} . \Gamma \frac{3}{2}$$

$$= 3^{\frac{3}{2}} . \frac{1}{2} . \Gamma \frac{1}{2}$$

$$= \frac{3^{\frac{3}{2}}}{2} . \sqrt{\pi}$$

BETA FUNCTION.

B(a,b) = $\int_0^1 x^{a-1} . (1-x)^{b-1}\, dx$

$$\frac{\Gamma a \,.\, \Gamma b}{\Gamma\, a+b}$$

Examples:

Evaluate each of the following integrals.

1. $\int_0^1 x^2\,(1-x)^3\,dx = \int_0^1 x^{3-1}\,.(1-x)^{4-1}\,dx$

$$\beta\,(3,4) = \frac{\Gamma 3\ \Gamma 4}{\Gamma 7} = \frac{2!\,.3!}{6!}$$

2. $\int_0^1 5x^3\ (1-x)^2\,dx = 5\int_0^1 x^{4-1}\,(1-x)^{3-1}\,dx$

$$= 5\beta\,(4,3) = 5\ \frac{\Gamma 4\ .\Gamma 3}{\Gamma 7}$$

$$= 5\,.\frac{3!\times 2!}{6!} = ??$$

3. $\int_0^1 2\,(1-x)^3\ dx = 2\int_0^1 x^{1-1}\,.(1-x)^{4-1}\,dx$

$$= 2\beta(1,4) \quad = 2.\frac{\Gamma 1\ \Gamma 4}{\Gamma 5} = \frac{1}{2}$$

Sets :

Difference between two events A and B = A-B is the events that consisting of all outcomes that are only in A.

A – B = $A \cap B'$ = event A occurs and event B does not occur.

B – A = $A' \cap B =$ event B occurs and event A does not occur.

$A' \cap B' =$ neither A nor B occur.

$A' \cup B =$ Event B or not event A occur.

$A \cup B' =$ Event A occurs or not event B occur.

$A' \cup B' =$ either not A or not B occur

Demorgan's laws:

$$(\overline{A \cup B}) = A' \cap B'$$

$$(\overline{A \cap B}) = A' \cup B'$$

Kinds of events:

Simple events: is an event that contains only one element(outcome. e.g. if a die is rolled once and a 6 turned up i.e. A is an event of appearing number 6 when the die is rolled only once.
A= {6} is a simple event.

Compound event: is an event that contains more than one element. e.g. a die is rolled once and the C is an event of getting an odd number, so
C= { 1, 3, 5} is a compound event.

Sure event: is an event that contains all outcomes of the sample space S. e.g. E is an event of getting a number when die is rolled one time, so
E = { 1,2,3,4,5,6}

Null or impossible event: is an event that does not contain any element (outcome) of the sample space, and is denoted by ϕ. e.g. the event of getting a letter when a die is rolled once. Or the event of getting a number when a coin is flipped.